PHYS 684.1
October 1, 2003

Homework #3

Due on October 8.

1. Read Edwards & Syphers section 3.3, and chapter 2 pp 18-41 and Familiarize yourself with the handout RHIC/AP/25, "Twiss Functions and Beam Sizes of the RHIC Insertion". Note the behavior of dispersion (η) between the arcs at the left and right ends of each plot.
2. Suppose that a particle traveling along the design orbit experiences an angular deflection θ at a location where the beta function is β_0. Show that downstream of this deflection, its motion is given by

$$x(s) = \theta \sqrt{\beta_0 \beta(s)} \sin\psi \tag{1}$$

   where $\psi = \psi(s) - \psi_0$ is the phase advance between the kick and measurement points.
3. Commonly beams are focused to small transverse sizes (and thus small β) at the center of detectors to maximize luminosity. Assume at the center of a detector, $\beta = \beta^\star$ and $\alpha = 0$. Show that:
   (a) At a drift distance s from the center of the detector (e.g. no fields), the beta function is given by $\beta(s) = \beta^\star + (s^2/\beta^\star)$.
   (b) The maximum possible phase advance within a drift is 180°.
4. The total counting rate of a physical process at a single collision point is $R = L\sigma$, where σ is the cross-section of the process, and the luminosity L [$\mathrm{cm}^{-2}\mathrm{s}^{-1}$] is an important measure of storage ring performance. When two bunches collide head on

$$L \;=\; 2fN_1N_2 \int \rho_1(x,y,s_1)\rho_2(x,y,s_2)dxdyd(\beta ct) \tag{2}$$

   where $s_1 = s + \beta ct$, $s_2 = s - \beta ct$, f is the collision frequency, N_1 and N_2 are the bunch populations, and ρ_1 and ρ_2 are the normalized distribution functions for the two bunches.
   (a) Using a Gaussian bunch distribution,

$$\rho(x,y,s) \;=\; \frac{1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_s} \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{s^2}{2\sigma_s^2}\right] \tag{3}$$

   where σ_x, σ_y, and σ_s, are the rms horizontal, vertical, and longitudinal bunch sizes, show that the luminosity for two bunches with identical distributions is

$$L \;=\; \frac{fN_1N_2}{4\pi\sigma_x\sigma_y} \tag{4}$$

   (b) Show that if the two bunches collide with a horizontal offset of b then the luminosity is reduced by a factor of $\exp(-b^2/4\sigma_x^2)$.